ΘΕΜΑ

Έστωσαν $X_1, X_2, \dots, X_{11} \sim N(0,\sigma^2)$

Να βρεθεί το c τέτοιο ώστε

$P\left(\left|\frac{S}{\bar{X}}\right| \le c\right) = 0{,}05$

ΛΥΣΗ

Είναι, $x_i \sim N(0,\sigma^2)$, $\forall i = 1,2,\dots,11 \Rightarrow \bar{X} \sim N\left(0,\frac{\sigma^2}{n}\right) \Rightarrow \frac{\bar{X}-0}{\sigma/\sqrt{n}} \sim N(0,1)$

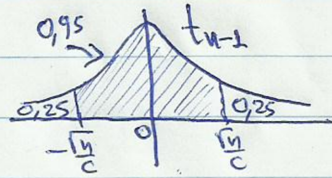
Προφανώς, $c>0$

$$P\left(\left|\frac{S}{\bar{X}}\right| \le c\right) = 0{,}05 \Rightarrow P\left(\left|\frac{\bar{X}}{S}\right| \ge \frac{1}{c}\right) = 0{,}05 \Rightarrow P\left(\left|\frac{\bar{X}}{S}\right| \le \frac{1}{c}\right) = 0{,}95 \Rightarrow$$

$$P\left(-\frac{1}{c} \le \frac{\bar{X}}{S} \le \frac{1}{c}\right) = 0{,}95 \Rightarrow P\left(-\frac{\sqrt{n}}{c} \le \frac{\bar{X}\sqrt{n}}{S} \le \frac{\sqrt{n}}{c}\right) = 0{,}95 \Rightarrow$$

$$P\left(-\frac{\sqrt{n}}{c} \le \frac{\bar{X}/\sigma/\sqrt{n}}{S/\sigma} \le \frac{\sqrt{n}}{c}\right) = 1-a \Rightarrow P\left(-\frac{\sqrt{n}}{c} \le \frac{\bar{X}/\sigma/\sqrt{n}}{\sqrt{\frac{(n-1)S^2}{\sigma^2}/n-1}} \le \frac{\sqrt{n}}{c}\right) = 0{,}95 \Rightarrow$$

$$P\left(-\frac{\sqrt{n}}{c} \le t \le \frac{\sqrt{n}}{c} \,\Big/\, t \sim t_{n-1}\right) = 0{,}95 \Rightarrow$$



$$2 \cdot P\left(0 \le t \le \frac{\sqrt{n}}{c}\right) = 0{,}95 \Rightarrow$$

$$P\left(0 \le t \le \frac{\sqrt{n}}{c}\right) = \frac{0{,}95}{2} \Rightarrow \overset{1/2}{\cancel{0{,}5}} - P\left(t \ge \frac{\sqrt{n}}{c}\right) = \frac{0{,}95}{2} \Rightarrow$$

$$\Rightarrow P\left(t \ge \frac{\sqrt{n}}{c}\right) = \frac{1-0{,}95}{2} = 0{,}25 \Rightarrow$$

$$\Rightarrow t_{0{,}25,\, n-1=10} = \frac{\sqrt{11}}{c} \Rightarrow c = \frac{\sqrt{11}}{t_{0{,}25,10}} = \frac{\sqrt{11}}{2{,}228} = 1{,}4886.$$